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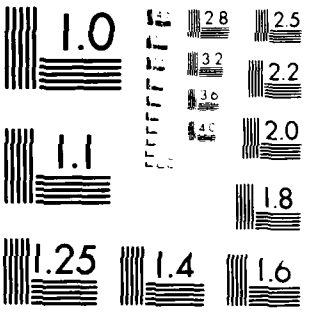

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NEW ADAPTIVE PROCESSOR FOR COHERENT SIGNALS AND INTERFERENCE

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ABSTRACT

In this paper we introduce a new adaptive processor able to work well even when the desired signal and the interference are coherent. The present adaptive processors fail to operate in these cases. The results of simulations appear to confirm the theoretical predictions.

I. Introduction

Since the pioneering work of Howells[1] , Applebaum[2] and Widrow[3] , there has been considerable activity in the development of adaptive beamforming for radar, sonar, communication, spectral estimation, *etc* .

Though the details differ in the different applications, the main assumptions and processing algorithms are essentially the same. In particular, a key assumption in all the previously cited work is that the interfering signals are not coherent with (i.e., do not have fixed phase differences from) the desired signal. More generally, two signal will be said to be coherent if one is a scaled and delayed replica of the other. Coherent interference can arise when multipath propagation is present, or when "smart" jammers deliberately introduce coherent interference, e.g. by retrodirecting the signal energy to the receiver. Coherence can completely destroy the performance of adaptive array systems.

In the reference[4,5] Gabriel and Evans *et al.* show that the subaperture sampling or spatial smoothing idea, as they call it, can be applied to the off-line eigenstructure based method for direction finding. This is an important contribution, the main idea of which we independently rediscovered later (see Shan *et al.* [6]). However, methods of applying this idea to adaptive beamforming (rather than direction finding) were not obvious. In this paper we will introduce an adaptive processor based on spatial smoothing algorithm.

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II. The Optimum Weight Vector of Adaptive Processor for Noncoherent and Coherent Inputs

It is easy to show that the optimum weight vector for different criteria is of the form

$$\mathbf{w}_{opt} = \alpha \mathbf{R}_{xx}^{-1}\mathbf{a}(\tau_0)$$

where $\mathbf{R}_{xx} = E\mathbf{x}(t)\mathbf{x}^*(t)$ is the covariance matrix of the array measurement vector $\mathbf{x}$, α is a scalar constant, and $\mathbf{a}(\tau_0)$ is so-called "look" direction vector.

We shall assume that we are interested in the signal $s(t)$ in the known look direction ϑ_0, and that the interfering signals are from the other K-1 unknown directions $\{\vartheta_1,\ldots,\vartheta_{K-1}\}$.

Noncoherent Signals

Under the assumptions of statistical independence and of noncoherence, we shall express the array measurement data vector as

$$\mathbf{x}(t) = \mathbf{a}(\tau_0)s(t) + \mathbf{A}\mathbf{j}(t) + \mathbf{v}(t)$$

where $\mathbf{A}$ specifies the interference directions

$$\mathbf{A} = [\mathbf{a}(\tau_1) \cdots \mathbf{a}(\tau_{K-1})]$$

and $\mathbf{s}(t)$, the array input signal vector, is consisting of desired signal $s(t)$ and jamming signals $\mathbf{j}(t)$

$$\mathbf{s}(t) = \begin{bmatrix} s(t) \\ \mathbf{j}(t) \end{bmatrix} = \begin{bmatrix} s(t) \\ j_1(t) \\ \vdots \\ j_{K-1}(t) \end{bmatrix} .$$

The covariance of $\mathbf{x}(t)$ can now be written

$$\mathbf{R}_{xx} = p_0^2\mathbf{a}(\tau_0)\mathbf{a}^*(\tau_0) + \mathbf{R}_{nn}$$

where

$$\mathbf{R}_{nn} = \mathbf{A}\mathbf{R}_{jj}\mathbf{A}^* + \sigma^2\mathbf{I}$$

and we have assumed, for simplicity, the noise intensity, σ^2, to be same at each sensor.

By using the matrix inversion lemma, we can show that

$$\mathbf{w}_{opt} = \beta \cdot \mathbf{R}_{nn}^{-1}\mathbf{a}(\tau_0) ,$$

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where β is a scalar constant.

We also introduce the modal representation

$$\mathbf{A}\mathbf{R}_{jj}\mathbf{A}^{*} = \sum_{i=1}^{K-1} \lambda_i \mathbf{e}_i \mathbf{e}_i^{*} ,$$

where $\{\lambda_i\}$ and $\{\mathbf{e}_i\}$ are the nonzero eigenvalues and the corresponding eigenvectors of the $M \times M$ matrix $\mathbf{A}\mathbf{R}_{jj}\mathbf{A}^{*}$, which will have rank $K-1$ because $\mathbf{R}_{jj}$ is the covariance matrix of the $K-1$ noncoherent signals.

Finally we shall also assume that the background measurement noise intensity is small compared to the signals $j(t)$, so that we shall have

$$\lambda_i \gg \sigma^2$$

and

$$\frac{1}{\sigma^2} \gg \frac{1}{\lambda_i + \sigma^2} .$$

Then we can write

$$\mathbf{w}_{opt} = \beta \mathbf{R}_{nn}^{-1} \mathbf{a}(\tau_0)$$

$$= \beta \left\{ \sum_{i=1}^{K-1} \frac{1}{\lambda_i + \sigma^2} \mathbf{e}_i \mathbf{e}_i^{*} + \sum_{i=K}^{M} \frac{1}{\sigma^2} \mathbf{e}_i \mathbf{e}_i^{*} \right\} \mathbf{a}(\tau_0)$$

$$\doteq \frac{\beta}{\sigma^2} \sum_{i=K}^{M} \rho_i \mathbf{e}_i \quad , \quad \rho_i = \mathbf{e}_i^{*} \mathbf{a}(\tau_0) ,$$

where $\doteq$ denotes asymptotic (as $\sigma^2 \to 0$) equality. Now by construction, the direction vectors $\{\mathbf{a}(\tau_1),\ldots,\mathbf{a}(\tau_{K-1})\}$ of the interfering signals, which are the columns of the matrix $\mathbf{A}$, lie in the span of the first $K-1$ eigenvectors $\{\mathbf{e}_1, \ldots, \mathbf{e}_{K-1}\}$ and are therefore **orthogonal** *to the remaining eigenvectors* $\{\mathbf{e}_K, \ldots, \mathbf{e}_M\}$.

Therefore, we shall have

$$\mathbf{w}_{opt}^{*} \mathbf{a}(\tau_l) \doteq \frac{\beta}{\sigma^2} \sum_{i=K}^{M} \rho_i \mathbf{e}_i^{*} \mathbf{a}(\tau_l) = 0 \quad l = 1,\ldots,K-1 ,$$

so that the beam pattern will have "deep nulls" in the interference directions. In the look direction, on the other hand, the constraint will ensure that we have

$$\mathbf{w}_{opt}^{*} \mathbf{a}(\tau_0) = 1 .$$

However, the situation deteriorates badly in the coherent case.

Coherent Signals

If the signals have *fixed* phase differences, which really means equal frequencies and fixed φ_i, then we shall have the representation

$$\mathbf{A}\mathbf{s} = \mathbf{a}(\tau_0) s(t) + \sum_{i=1}^{K-1} \mathbf{a}(\tau_i) j_i(t)$$

$$= [\mathbf{a}(\tau_0) + \gamma_1 \mathbf{a}(\tau_1) + \cdots + \gamma_{K-1} \mathbf{a}(\tau_{K-1})] s(t)$$

where the $\{\gamma_i\}$ are fixed complex constants given by

$$\gamma_i = (p_i / p_0)\, e^{j(\varphi_i - \varphi_0)} \quad , \quad i = 1,\ldots,K-1$$

In this case, the covariance matrix, $\mathbf{A}E\mathbf{s}\mathbf{s}^{*}\mathbf{A}^{*}$, will have rank 1, so that it will have only one nonzero eigenvalue λ_1, and the covariance matrix R_{zz} will have $M-1$ eigenvalues equal to σ^2.

Therefore we shall have

$$\mathbf{w}_{opt} = \alpha \mathbf{R}_{zz}^{-1} \mathbf{a}(\tau_0)$$

$$= \alpha \left[\frac{1}{\lambda_1 + \sigma^2} \mathbf{e}_1 \mathbf{e}_1^{*} + \sum_{i=2}^{M} \frac{1}{\sigma^2} \mathbf{e}_i \mathbf{e}_i^{*} \right] \mathbf{a}(\tau_0)$$

$$\doteq \alpha \sum_{i=2}^{M} \frac{1}{\sigma^2} (\mathbf{e}_i^{*} \mathbf{a}(\tau_0)) \mathbf{e}_i$$

All we can say here is that the linear combination

$$\mathbf{b} := \mathbf{a}(\tau_0) + \gamma_2 \mathbf{a}(\tau_1) + \cdots + \gamma_K \mathbf{a}(\tau_{K-1})$$

will be orthogonal to the $\{\mathbf{e}_2, \ldots, \mathbf{e}_M\}$. This does not, however, imply that the same will be true of the $\{\mathbf{a}(\tau_1),\ldots,\mathbf{a}(\tau_{K-1})\}$ individually, and therefore there will not in general be any nulls in the directions of the interfering signals.

Due to the constraint, we shall have

$$\mathbf{w}_{opt}^{*} \mathbf{a}(\tau_0) = 1 ,$$

but this is of small comfort, because the actual array output will be

$$y(t) = \mathbf{w}_{opt}^{*} \mathbf{x}(t)$$

$$\doteq \alpha \left[\sum_{i=2}^{M} \frac{1}{\sigma^2} (\mathbf{e}_i^{*} \mathbf{a}(\tau_i)) \mathbf{e}_i^{*} \right] [\mathbf{b} s(t) + v(t)]$$

where we recall that $\mathbf{b} := \mathbf{a}(\tau_0) + \sum_{i=2}^{M} \gamma_i \mathbf{a}(\tau_i)$ lies along $\mathbf{e}_1$ and is orthogonal to $\{\mathbf{e}_2, \ldots, \mathbf{e}_M\}$. Therefore there will be no signal output from the conventional array when the signals are coherent.

A Way Out

This analysis also makes clear what is necessary to rescue the situation: we must somehow restore the rank of the covariance matrix $E(\mathbf{A}\mathbf{s})(\mathbf{A}\mathbf{s})^{*}$ to being K. Then the noise-alone eigenvectors will be orthogonal to all the vectors in the space of the signals and the beam pattern will

have nulls in the directions of the interfering signals. A simple scheme for achieving this rank restoration with an adaptive algorithm is proposed in the next section.

III. A New Adaptive Array Processor

We shall describe a preprocessing scheme for the sensor outputs that will restore the rank of the input signal covariance matrix to K even if the inputs are completely coherent with each other.

The scheme is based on combining measurements from overlapping subarrays.

Given the M sensor outputs at any time instant,

$$\mathbf{x}(t) = [x_1(t) \cdots x_M(t)]^T ,$$

define p subsets (recall that K is the number of sources)

$$\mathbf{z}^{(1)}(t) = [x_1(t) \cdots x_{K+1}(t)]^T$$

$$\mathbf{z}^{(2)}(t) = [x_2(t) \cdots x_{K+2}(t)]^T$$

$$\vdots$$

$$\mathbf{z}^{(p)}(t) = [x_p \cdots x_{K+p}(t)]^T .$$

Define

$$\mathbf{R}_{zz}^{(k)} = E\mathbf{z}^{(k)}\mathbf{z}^{(k)*}$$

and the spatial smoothed correlation matrix:

$$\bar{\mathbf{R}} = \frac{1}{p}\sum_{k=1}^{p} \mathbf{R}_{zz}^{(k)}$$

Then we can prove (see Reference[7,8]) that $\bar{\mathbf{R}}$ will have the form

$$\bar{\mathbf{R}} = \mathbf{A}\bar{\mathbf{S}}\mathbf{A}^* + \sigma^2 I$$

where

$\bar{\mathbf{S}}$ will have rank K if and only if $p \geq K$.

Once $\bar{\mathbf{S}}$ has rank K, then the noise eigenvectors will be orthogonal to the columns of $\mathbf{A}$ and by the analysis of Section III, will give nulls in the interference directions.

Fig.1a shows how to form the spatial data subset from one 'snapshot' A flow diagram of the proposed adaptive processor is shown in Fig.1b. At each time instant, the 'snapshot' of M data samples is divided into overlapping subgroups of K samples each, these subgroups are then fed in succession into the adaptive processor, which updates a K-dimensional (if each sensor is followed by a N tapped-delay-line, then a $K \times N$ -dimensional) weight vector each time. After all the subgroups have been processed, the same procedure is repeated with the next 'snapshot'.

It is important to note that the processor can be any kind of adaptive processor using any adaptive algorithm and any array structure, e.g. the Howells-Applebaum or the Frost arrays could be used. There are several ways to form the array output and they have deferent effects on the performance of the processor. Further studies on this subject are being conducted.

IV. The Computer Simulation Results

In the computer simulation example, we have a signal $0.1 sin 0.4\pi t$ arriving at 90° and two coherent interfering signals arriving at 50° and 130°. Fig 2a shows the beampattern of a conventional Frost array with six sensors. Our new scheme uses 10 sensors with subarray of size six, adapted with the same LMS algorithm, and gives the beampattern shown in Fig.2b. The input signal waveform is shown in Fig.3a, with the output of the conventional array in Fig.3b and that of new processor in Fig.3c - notice the big difference in the scales of these figures. The results would seem to speak themselves.

Several other simulations have shown similar results.

V. Conclusions

Conventional adaptive processors perform very poorly in coherent receiving environments. If the received signal is coherent with one interference, the signal will be canceled out on the output, the processor will therefore totally fail. The suggested new adaptive processing system is able to overcome this degradation of performance in coherent input environments, without considerably increasing the complexity of the system structure or the computational burden. The new array structure can be applied in conjunction with any of the adaptive algorithms and structures of current adaptive arrays, and successfully separates the coherent array inputs, as shown by theoretical analysis and simulation results.

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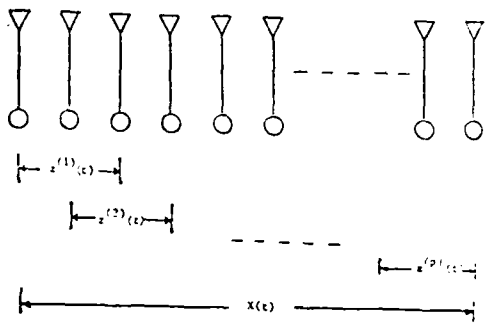

Figure 1a: Spatial Smoothing

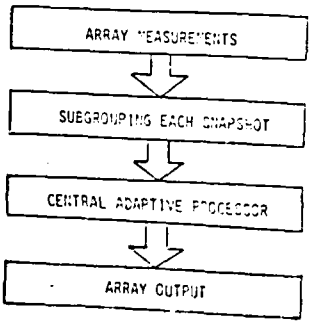

Figure 1b: Flow Diagram of The New Processor

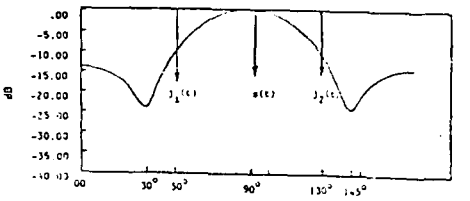

Figure 2a: Beampattern of the Frost Array With Coherent Inputs

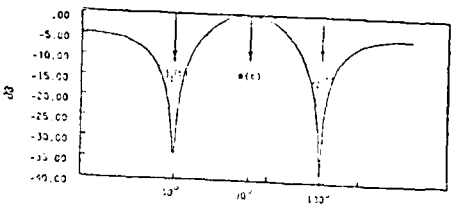

Figure 2b: Beampattern of the New Processor With Coherent Inputs

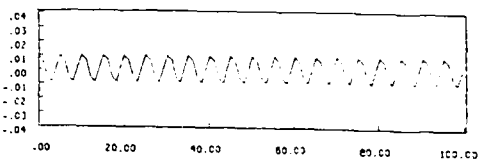

Figure 3a: Input Signal
$s(t) = 0.1 \sin 0.4\pi t$

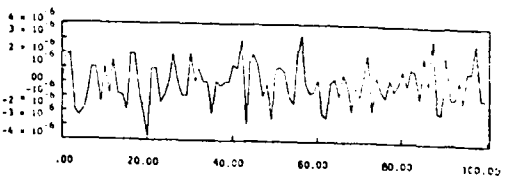

Figure 3b: Output of Frost Array With Coherent Interference

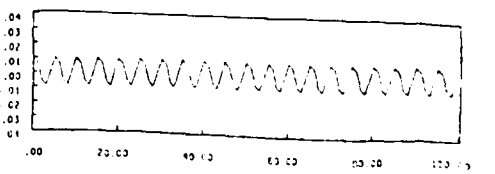

Figure 3c: Output of the New Processor With Coherent Interference

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